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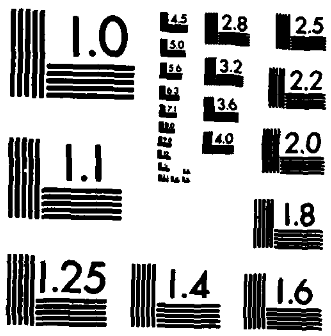

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Technical Report 607

S.L. Bernstein

Multiple Resource Replenishment with Multi-Mission Satellite Applications

2 September 1982

Prepared for the Department of the Air Force
under Electronic Systems Division Contract F19628-80-C-0002 by

Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

Thomas J. Alpert

Thomas J. Alpert, Major, USAF
Chief, ESD Lincoln Laboratory Project Office

ESD-TR-82-081

AD-A121 009

ERRATA SHEET
for
TECHNICAL REPORT 607

Errors have been found on pages 29 and 30 of MIT/LL Technical Report-607 ("Multiple Resource Replenishment with Multi-Mission Sateliite Applications", by S. L. Bernstein, dated 2 September 1982). Please make the following corrections in your copy.

Page 29

The next to the last line should read:
"b. $(E_A + E_B)/N_O$ is the expected normalized excess..."

Page 30

The vertical axis of Figure IV.7(b) should be labeled "$(E_A + E_B)/N_O$" and the "0.5" tick should be changed to "0.6".

9 November 1982

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

MULTIPLE RESOURCE REPLENISHMENT WITH MULTI-MISSION SATELLITE APPLICATIONS

S.L. BERNSTEIN

Group 68

TECHNICAL REPORT 607

2 SEPTEMBER 1982

LEXINGTON MASSACHUSETTS

ABSTRACT

Both scheduled and replenishment as-needed strategies are analyzed for multiple resource systems. It is found that, in general, the most efficient strategies are those for which the replenishment is made with a combination of units most closely resembling the failed resources. In this context, "most efficient" refers to the smallest expected unit replenishment rate required to generate a given level of continuing service. Quantitative results are given for many cases of interest including replenishment with multiple resources of different types. The techniques used can be extended to other replenishment strategies and failure models.

The results are particularly applicable to multi-mission satellite systems and can contribute to the economic analysis of such systems.

CONTENTS

LIST OF ILLUSTRATIONS

I. INTRODUCTION

This report examines several topics relating to the replenishment of multiple resources. The general problem has the following outline: It is desired to maintain S systems in operation. In order to be considered operational the s-th system must have $N_o(s)$ working units. However, units fail in a random, but describable way, thus requiring replenishment. Several models for describing unit failures will be considered as will several replenishment strategies.

This class of problem was motivated by considering the particular case of multi-mission satellites. Using satellite terminology the "units" of a system are payloads that must be maintained on orbit. A system will also be referred to as a "mission". Replenishment is accomplished by launching additional satellites. Each satellite may carry multiple payloads to support the multiple missions. Each satellite consists of a "bus" with support equipment such as stabilization control, power supplies, etc., in addition to the multiple payloads. It is assumed that each on-orbit payload is either working satisfactorily or has failed. If the bus fails on a particular satellite, all the payloads on that satellite are considered as having failed. Thus multiple simultaneous failures will be considered in addition to individual payload failures.

This is illustrated in Figure I-1a, b, c for S=3 missions. Also shown are the options of launching satellites containing 1, 2 or 3 payloads each. Note that only an initial deployment is represented. After failures have occurred there may be satellites in orbit with less than their initial number of operating payloads. Replenishment satellites could also have 1, 2 or 3 payloads each.

A major issue in the system design is the selection of the best number of payloads per satellite. A key factor in answering that question is the replenishment launch rate. As will be seen subsequently, the rate of payload replenishment, e.g., payloads per year, increases with the number of payloads per satellite. This increase comes about from a mismatch between the minimum

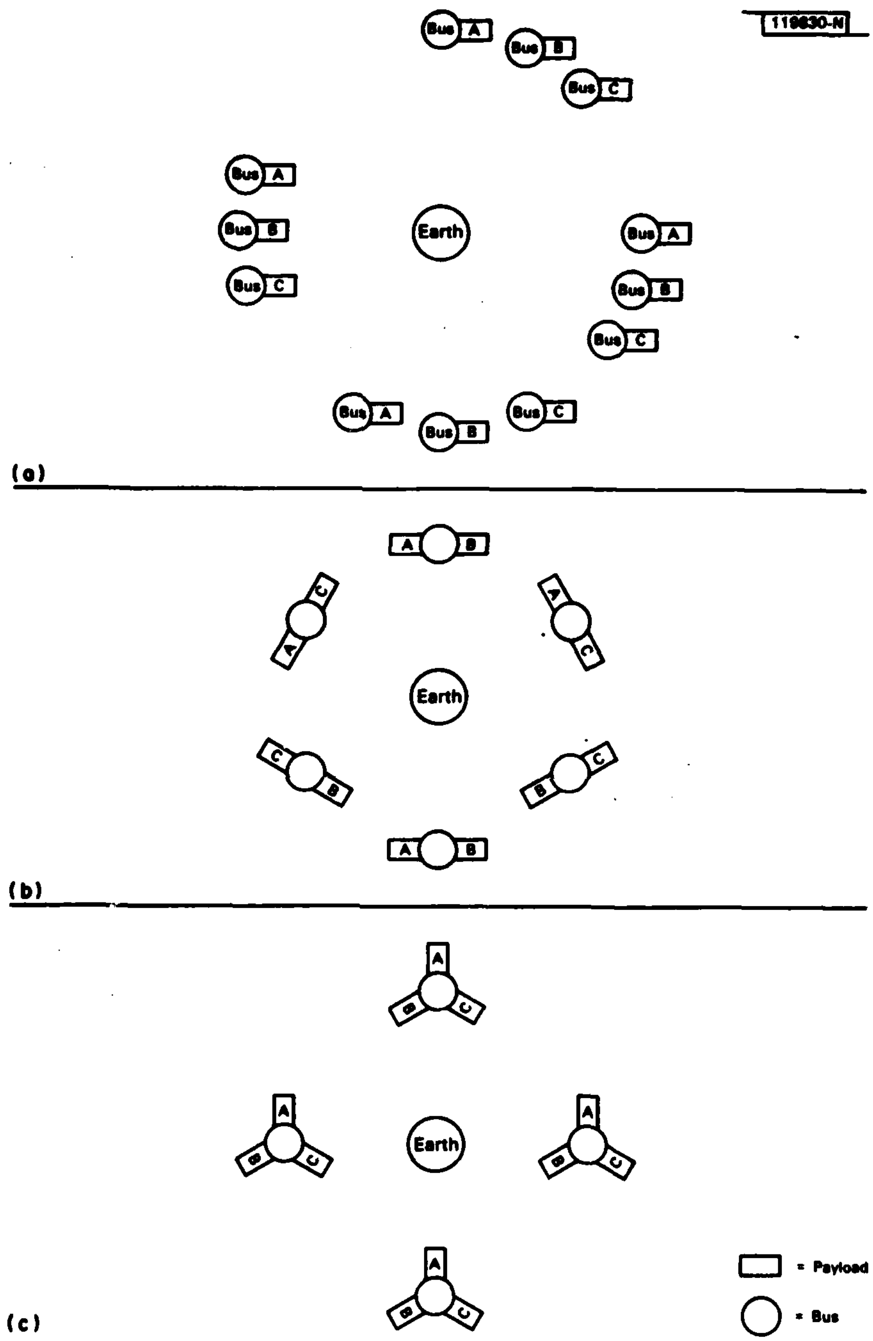

Fig. I.1. Multi-mission satellite systems (S=3, N_o=4).

replenishment required (e.g., only Mission A) and the capabilities of the replenishment satellite (e.g., Missions A and B). This increase could be balanced by the possibility of a reduced cost per payload pound of multi-mission satellites due to shared bus functions. (Also entering into the decision would be the increased cost per pound of developing such satellites.) This paper will not consider these economic issues further and will only address the replenishment launch rate issues.

The results to be presented are felt to be applicable to situations other than maintaining satellite systems on-orbit. For example, consider a grocer trying to maintain N_o boxes of S types of junk food on his shelves. He might have a choice of a large number of small (expensive per box) deliveries filling in just what he needs or a smaller number of larger (cheaper per box) deliveries. He might also have the choice of ordering when running low (replenishment as needed) or having a pre-determined delivery quantity (scheduled replenishment). All of these replenishment strategies will be treated.

The next section treats the case of scheduled replenishments, i.e., replenishment strategies that do not rely on the status of the systems.* These results are applicable to arbitrary S (number of resources). When considering the more efficient strategies of replenishing when needed (including bulk replenishment of multiple missions even though only one requires it) the analysis is a good deal more involved. The cases of S = 1,2,3 are treated in separate sections. An attempt will be made to unify results as they accumulate.

* However, they are important to consider because it is not always possible to arrange satellite launches on an "as-needed" basis; launch dates are often set years in advance.

II. SCHEDULED REPLENISHMENT

A. General Considerations

"Scheduled replenishment" refers to strategies by which resources are replenished according to some scheduling rule without regard to the actual amount of resource needed. One expects such strategies to result in required replenishment rates greater than those required by "replenish as needed" strategies.

The basic results of this section apply to an arbitrary number of systems (resources) S because the replenishment rates to be found are really <u>per system</u>.

It can be shown in general that the <u>average</u> number of units that will be found in service, $\overline{N}$, is given by:

$$\overline{N} = \lambda \overline{L} \tag{II-1}$$

where

λ = average replenishment rate (units/time period).

$\overline{L}$ = average lifetime of a unit.

This intuitive relationship is an application of Little's result [Ref. 1, pg 17] and is valid for most replenishment and failure statistics that are stationary with time.

If it is desired to maintain N_o units in operation with fairly high probability then the average replenishment rate will need to be greater than $N_o / \overline{L}$ because Eq. II-1 shows that this rate would yield only N_o working units <u>on average</u>. The next sections explore this issue for representative cases. One further definition will be introduced now that will be used throughout:

$$\mu \triangleq 1/\overline{L} \tag{II-2}$$

= each unit's failure rate (per time period)

Further physical interpretation for μ will be given in Section C.

B. Periodic Replenishment

Several years ago C. Niessen[2] studied the periodic replenishment problem. He considered units that had either expontential or uniform distributions for their lifetimes. (He also considered launch failures as well as the replenishment of more than one identical unit at a time.) A brief summary of his technique follows for the case of periodic, single-unit, successful replenishments:

After the K-th launch at time (K-1)T the number of surviving units N(t) can be represented as a sum of independent random binary variables:

$$N(t) = \sum_{j=0}^{K-1} W\,(t-jT) \qquad \text{(II-3)}$$

where T = time between launches and

$$W(t) = \begin{cases} 1 \text{ if unit launched t-ago still survives} \\ 0 \text{ if unit launched t-ago has failed} \end{cases}$$

For exponentially distributed unit life-times

$$P\,[W(t) = 1] = \begin{cases} e^{-\mu t} & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad \text{(II-4)}$$

and for uniformly distributed life-times

$$P\,[W(t) = 1] = \begin{cases} 1-\frac{\mu t}{2} & 0 < t < \frac{2}{\mu} \\ 0 & \text{elsewhere} \end{cases} \qquad \text{(II-5)}$$

He was then able to calculate the probability distribution of N(t). An example is shown in Figure (II-1) for the particular case of exponential life-times and N=4 units surviving on average. (In this particular example the probability of successful launch is 0.85. The expressions in II-4 and

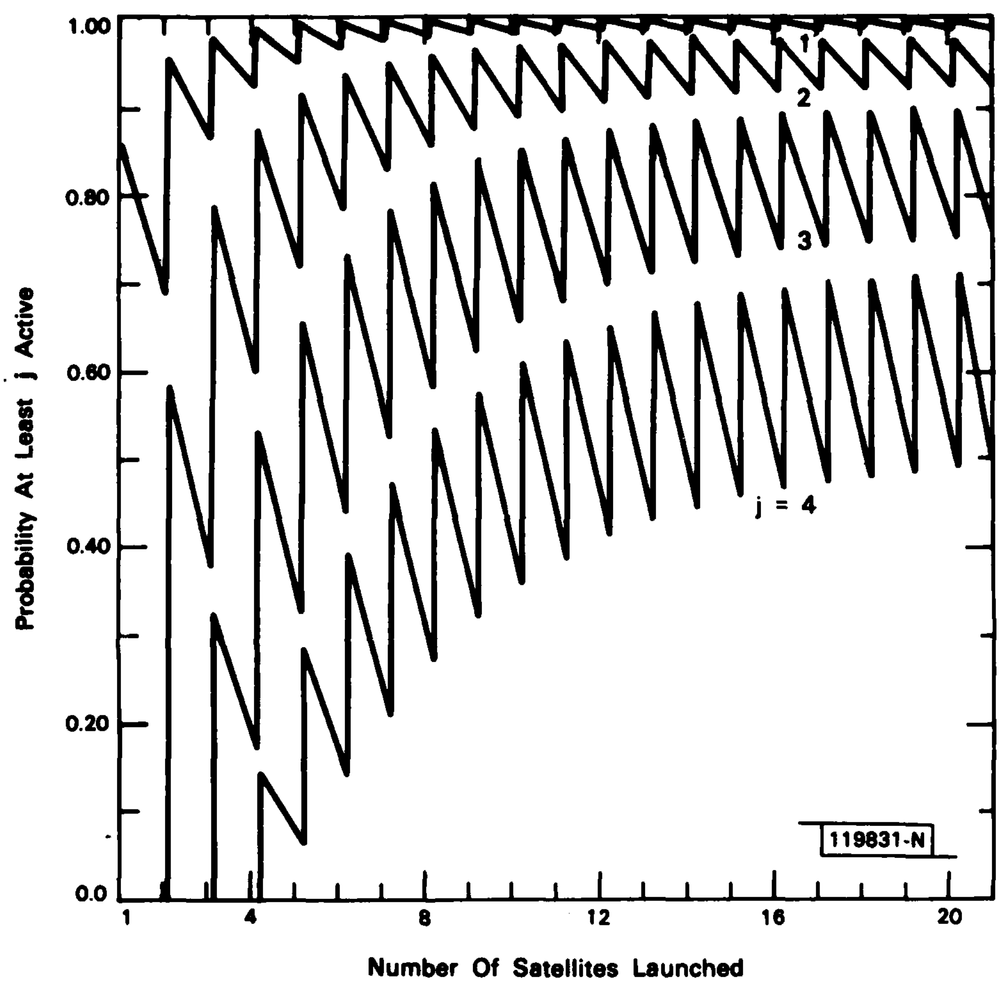

Fig. II.1. Survival probabilities for periodic replenishment. Average number of active units = 4. Probability of successful launch = .85. From Niessen, Ref. [2].

II-5 can be multiplied by 0.85 to take this into account.) The sawtooth nature of the curves reflects the periodic launch strategy. Note that in equilibrium the probability of 4 or more units operating successfully drops to about 0.5 just before each launch. If the desired number of operational units were 4 this would not be reliable enough. Figure (II-2) also taken from [2] but with $\bar{N}$=6 shows that this 50% increase in launch rate would yield a probability > 0.8 of maintaining 4 operational units. As will be discussed in Section III, this is 50% above the replenishment rate that would be required for launching only when needed.

C. Poisson Replenishment/Exponential Lifetime

An easier model to work with and one which also could be realistic in many cases is that of Poisson replenishment scheduling and exponentially distributed lifetimes for each unit. The Poisson replenishment scheduling physically (and roughly) means that replenishment can take place during any small interval of time independent of previous or future events. This model leads to exponentially distributed inter-replenishment times. The same interpretation holds for unit failures with exponentially distributed lifetimes. A more complete description of this system is as follows.

Let

$$\begin{aligned} &\lambda = \text{average replenishment rate} \\ \text{and} \quad &\mu = 1/\bar{L} \\ &\quad = \text{average failure rate per unit.} \end{aligned} \tag{II-6}$$

(Both of these rates are with respect to time.) That is, the probability of replenishing a unit in a small time interval Δt will be $\lambda(\Delta t)$ independent of time* and independent of the number of surviving units. The probability that a unit survives an interval of time t is $e^{-\mu t}$ and its probability of failure is $\mu(\Delta t)$ in interval Δt. The unit's average lifetime $\bar{L}$ is equal to

* The replenishment rate could advantageously be made dependent on time if the system's state were known. This strategy can be analyzed but will not be pursued further.

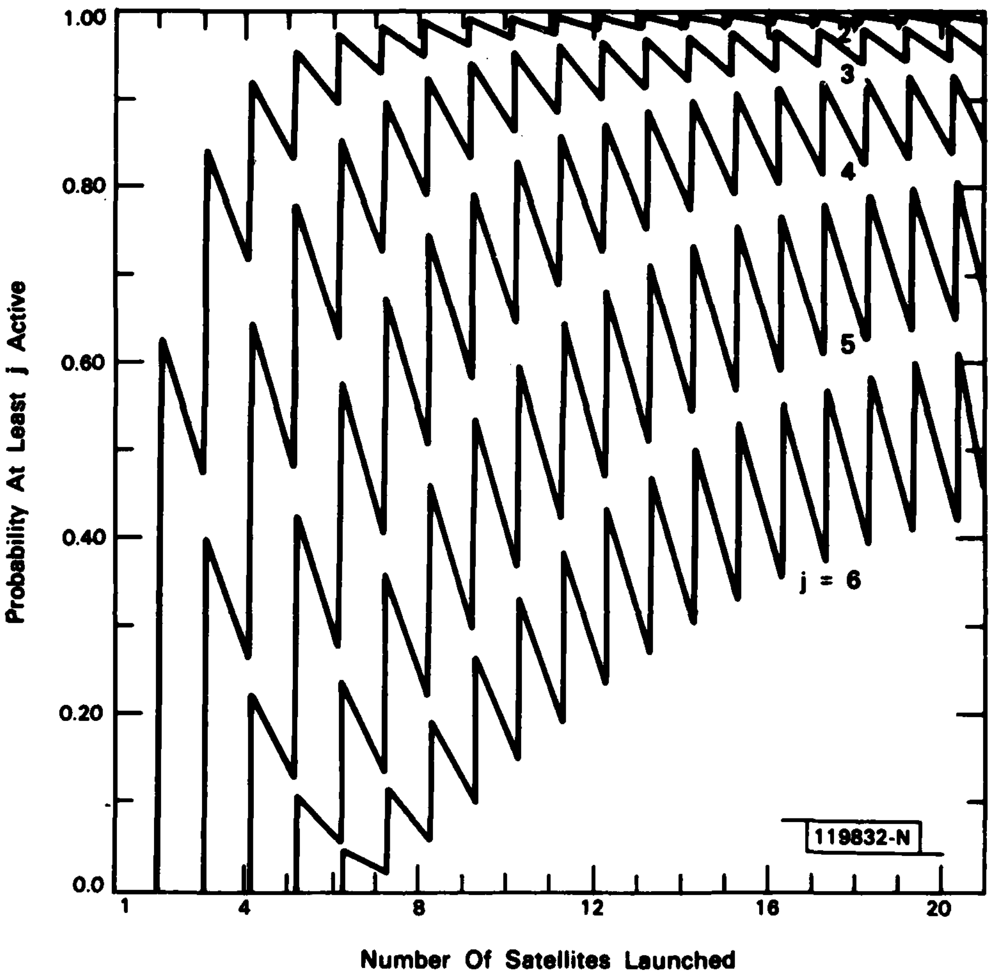

Fig. II.2. Survival probabilities for periodic replenishment. Average number of active units = 6. Probability of successful launch = 0.85. From Niessen, Ref [2].

$1/\mu$. If N units are operational at any time then the probability of a failure in time Δt is $N\mu(\Delta t)$. The probability of more than one failure in Δt is negligible.

This replenishment strategy can be represented by the continuous time Markov chain shown in Figure II-3. (Kleinrock, Reference 1, Chapters 2-4 provides a good reference for all of the Markov process manipulations to be performed.) Each circle represents a state of the system which in this case is chosen to be the number of surviving units. The arrows represent transition rates which if multiplied by Δt would give the probability of making the indicated transition in time Δt.

1. Equilibrium Solution

This particular chain is seen to be a birth-death process with a well known equilibrium solution: The probability, P_j, of being in state j (that is, having j units in operation) is:

$$P_j = \frac{(\lambda/\mu)^j}{j!} e^{-(\lambda/\mu)} \qquad j \geq 0 \qquad \text{(II-7)}$$

which is simply a Poisson distribution.* The average number of units in operation will be

$$\begin{aligned} \overline{N} &= \sum_{j=0}^{\infty} j \quad P_j \qquad \text{(II-8)} \\ &= \lambda/\mu \end{aligned}$$

which is a special case of Eq. (II-1). It is also well known that the variance of the number of units in operation will be:

$$\sigma_N^2 = \lambda/\mu \qquad \text{(II-9)}$$

which is numerically equal to the average number.

* This result is valid for arbitrarily distributed lifetimes.

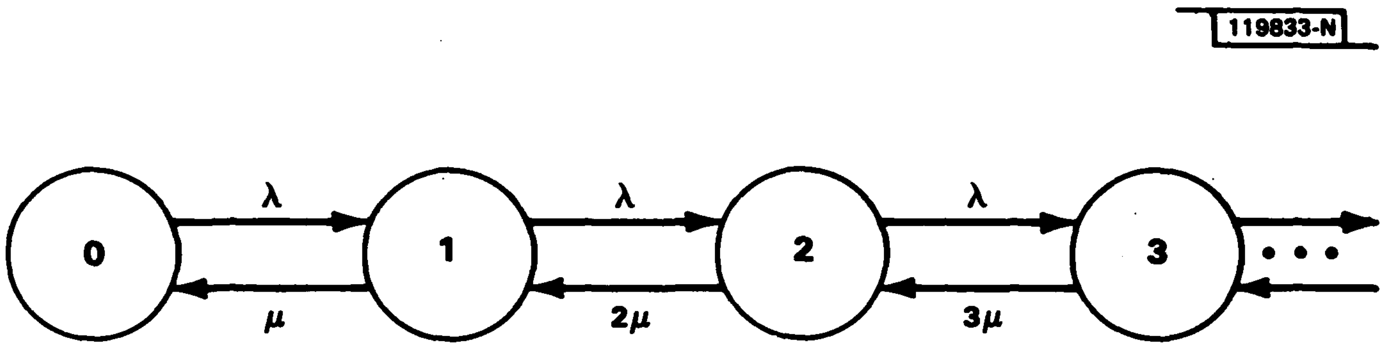

Fig. II.3. State diagram representing Poisson replenishment and exponentially distributed unit lifetimes.

Figure II-4 shows the replenishment rates needed to maintain at least N_o operating units with various degrees of confidence. The launch rate is expressed as the normalized quantity

$$\frac{\lambda}{N_o \mu} = \frac{\overline{N}}{N_o} = \frac{\text{average number of units in operation}}{\text{number of units required}} \qquad \text{(II-10)}$$

The actual launch rate required, λ, increases with N_o and μ, and depends on the confidence desired. The confidence is expressed as the probability that at least N_o units are in operation. The figure shows that as N_o increases the required $\overline{N}$ approaches N_o. This happens for large numbers of units because the relative fluctuation of the number of units around the mean is small and a replenishment rate about equal to the failure rate ($N_o \mu$) will suffice. At low or moderate N_o, however, replenishing at the average failure rate is not sufficient. For example, to keep at least N_o=4 units in operation with probability .9 will require a replenishment rate sufficient to keep $\overline{N}$ = 6.7 units in operation on average. (Note: If it were desired to account for unsuccessful replenishments, the rate λ should be considered as the rate of successful replenishments only. The replenishment process formed by independent failures of exponentially scheduled replenishments still has an exponential distribution.)

2. Transient Solution

Figures II-1 and II-2 show the transient build-up to steady-state for two examples of periodic replenishment. The transient solution for Possion replenishment and exponential life-times exhibits similar behavior, but with an easily attained closed form.* In particular, if units are begun to be put into operation at time t=0 at rate λ, then at time t, the average number in service, $\overline{N}(t)$, will be:

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* See, for example, Kleinrock [Ref. 1], page 82, problem 2.12.

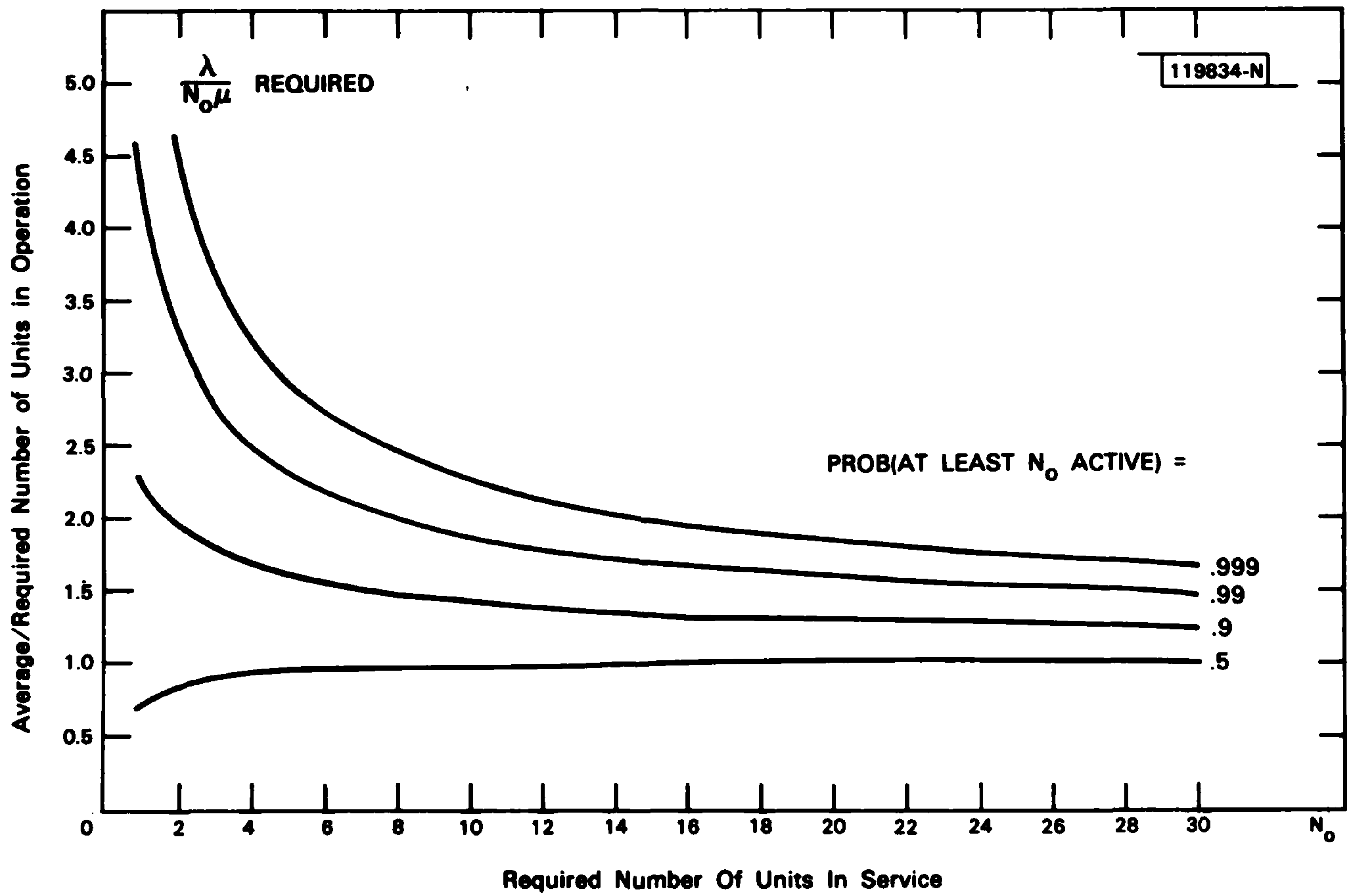


Fig. II.4. Required average replenishment rate (Poisson replenishment, exponential lifetime).

$$\overline{N}(t) = \frac{\lambda}{\mu}(1-e^{-\mu t}) \tag{II-11}$$

and the probability that j are in service will be:

$$P_j(t) = \frac{[\overline{N}(t)]^j}{j!} e^{-\overline{N}(t)} \tag{II-12}$$

which is seen to be a time-varying Poisson distribution with Eq.(II-7) giving the steady-state values. The form for $\overline{N}(t)$ shows that equilibrium is reached with a time constant of $1/\mu$, depending only on the lifetime of each unit.

If there were K_o units in service at time t=0, the number of these original units serving to time t would follow a binomial distribution with:

$$P(\text{k units surviving out of } K_o) = \binom{K_o}{k} e^{-\mu tk}(1-e^{-\mu t})^{K_o-k} \tag{II-13}$$

Thus, the number of units in service at time t, given K_o were in service at t=0 with Poisson replenishment thereafter is the sum of a binomial and a Poisson random variable. Using discrete convolution, the complete solution to the probability of finding j units in service at time t is:

$$P(\text{j units at time t} \mid K_o \text{ at } t=0)$$

$$= \sum_{k=0}^{j} \binom{K_o}{k} e^{-\mu tk}(1-e^{-\mu t})^{K_o-k} \frac{[\overline{N}(t)]^k}{k!} e^{-\overline{N}(t)} \tag{II-14}$$

III. REPLENISHMENT AS-NEEDED, S=1 SYSTEM.

To begin the discussion of replenishment-as-needed strategies, the simple case of S=1 system will be considered. Only exponential lifetime models will be treated and it will be assumed that replenishment takes place instantaneously.*

Suppose it is desired to maintain N_o units in service, each unit having failure rate μ (average lifetime $1/\mu$). The required replenishment rate must be equal to the rate of failure generation. The rate of failure generation will be $N_o\mu$, N_o times the failure rate of one unit. Again, letting λ = the replenishment rate, the fundamental relationship is:

$$\lambda = N_o\mu \qquad \text{(III-1)}$$

A proper derivation of III-1 is as follows: Beginning at any time, a replenishment will next be required when any of the N_o units in service fails. The probability that no replenishment will be required in the next time period of duration t will be:

$$P\text{ (no replenishment in next } t) = [e^{-\mu t}]^{N_o} \qquad \text{(III-2)}$$

which is the probability that all N_o units have survived. (Note that the memoryless property of the exponential distribution is being fully exploited.) If the probability density function of the time to next replenishment is denoted as $p_r(t)$, then Eq. III-2 says:

---

* In practical terms, it is assumed that replenishment is accomplished in a time much shorter than the average time to the next failure. In the satellite case the number of active units required, N_o, usually includes an on-orbit spare to provide continuity of service. It is implicitly assumed that there is an inventory of units which will always be able to supply a replenishment unit when needed.

$$\int_0^t p_r(x)\,dx = 1- e^{-N_o \mu t} \qquad \text{(III-3)}$$

and hence

$$p_r(t) = N_o \mu\, e^{-N_o \mu t} \qquad \text{(III-4)}$$

It is seen immediately from (III-4) that the time to next replenishment is exponentially distributed with average value $1/(N_o \mu)$. This not only verifies (III-1) as giving the average rate of replenishment, but shows that the replenishment process is Poisson. Specifically, the probability of requiring j replenishments in any time duration T, $P_j(T)$ will be:

$$P_j(T) = \frac{(N_o \mu T)^j}{j!} \quad e^{-N_o \mu T} \qquad \text{(III-5)}$$

The average number required will be $N_o \mu T$ and the standard deviation will be $\sqrt{N_o \mu T}$. Thus, taken over a time period long enough to average 100 replenishments the standard deviation will be only 10% of the average. However, the relative fluctuation in the number of replenishments will be much larger over time durations averaging only, say, 10 or 20 replenishments. For example, if the expected number of replenishment over some time period is 10, then there is a 21% chance of requiring 13 or more replenishments which is 30% more than the average. Satellite systems for which the operational usefulness lasts through only 10 or so replenishments must expect considerable uncertainty as to the total number of satellites to be procured.

IV. REPLENISHMENT AS-NEEDED, S=2 SYSTEMS

In order to analyze the replenishment as-need strategy when S=2 systems are involved, it should first be noted that if each system is replenished separately, e.g., one payload per satellite, then the results of the previous section apply directly.

The interesting case here assumes that the units are replenished two-at-time with one unit of each type, i.e., no "doubles". If the two systems are labeled A and B, then if either system requires replenishment it will be assumed that an A and B unit will be provided even though one of them is not needed at that moment. (In the satellite case this is equivalent to saying that all launched satellites are configured with both payloads A and B.) Thus while one system has exactly N_o units in operation, the other one will have at least N_o. (Only the case where equal numbers of A and B units are required will be considered here. The models to be presented would generalize to unequal requirements in a straightforward way.)

Two failure models will be considered:

Independent failures of all units

and

Dependent failures corresponding to the loss of a satellite bus in addition to independent failures.

A. Independent Failures

As applied to the satellite case, independent unit failures correspond to considering only payload failures, ignoring the possibility of bus failures or assuming that the bus failure rate is much less than that for the payloads. This is illustrated in Figure IV-1 for $N_o = 4$ which shows satellites carrying packages only. In some cases, the satellites have a failed package indicated by a blank. Replenishment satellites always carry an A and B package. While there are only 4 active A packages, the example shows two "excess" B's which are still active.

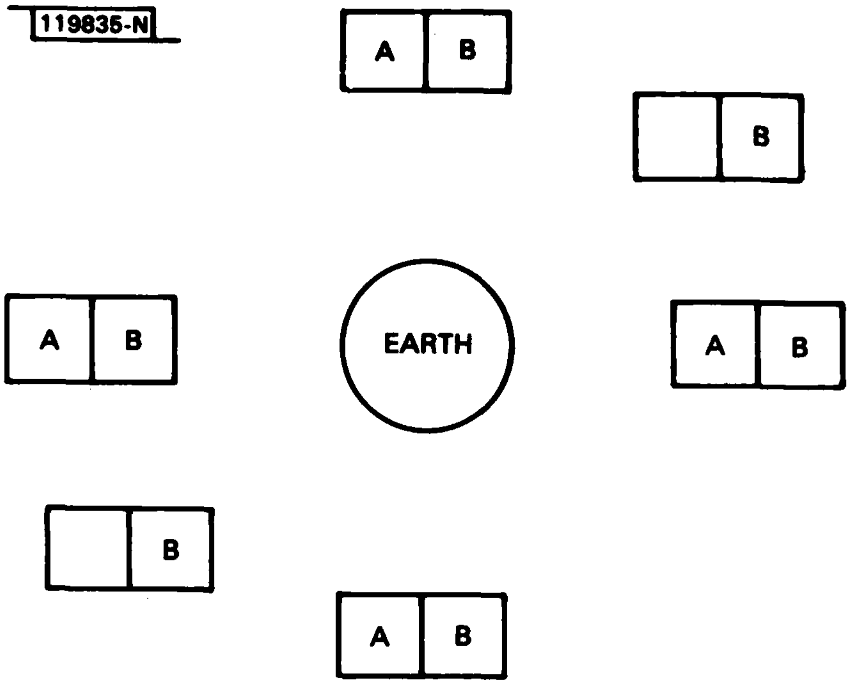


Fig. IV.1. Two satellite systems, $N_o = 4$ active units required of each (two excess B's shown).

Figure IV-2 shows how the replenishment system can be modeled as a Markov Chain on a 2-dimensional grid. The horizontal axis gives the number of active A units and the vertical gives the number of active B units. The only allowed states are along the L-shaped path shown where one or both systems must have exactly N_o active units. The failure rates for each A and B unit are μ_A and μ_B respectively. Note that the chain is driven to the right ("excess" A's) by failures in the N_o B units which are active and driven upwards by failures in the A units.

The state probabilities*, P(j,k), are readily found by using the well-known techniques applied to one-dimensional birth-death queues in equilibrium [1, Chapter 3]. In particular, the rate of crossing a dividing line between two states must be equal in both direction while in equilibrium.
For example,

$$\underbrace{(N_o+j)\mu_A \quad P(N_o+j,\, N_o)}_{\text{rate left}} = \underbrace{N_o\mu_B P(N_o+j-1,\, N_o)}_{\text{rate right}} \qquad j>0 \tag{IV-1}$$

Using $P(N_o, N_o)$ as the starting point gives:

$$P(N_o+j,\, N_o) = \frac{(N_o\mu_B/\mu_A)^j}{\prod_{k=1}^{j} (N_o+k)} P(N_o,\, N_o) \qquad j>0 \tag{IV-2a}$$

and

$$P(N_o,\, N_o+j) = \frac{(N_o\mu_A/\mu_B)^j}{\prod_{k=1}^{j} (N_o+k)} P(N_o,\, N_o) \qquad j>0 \tag{IV-2b}$$

* That is, the probability of having j A units and k B units active.

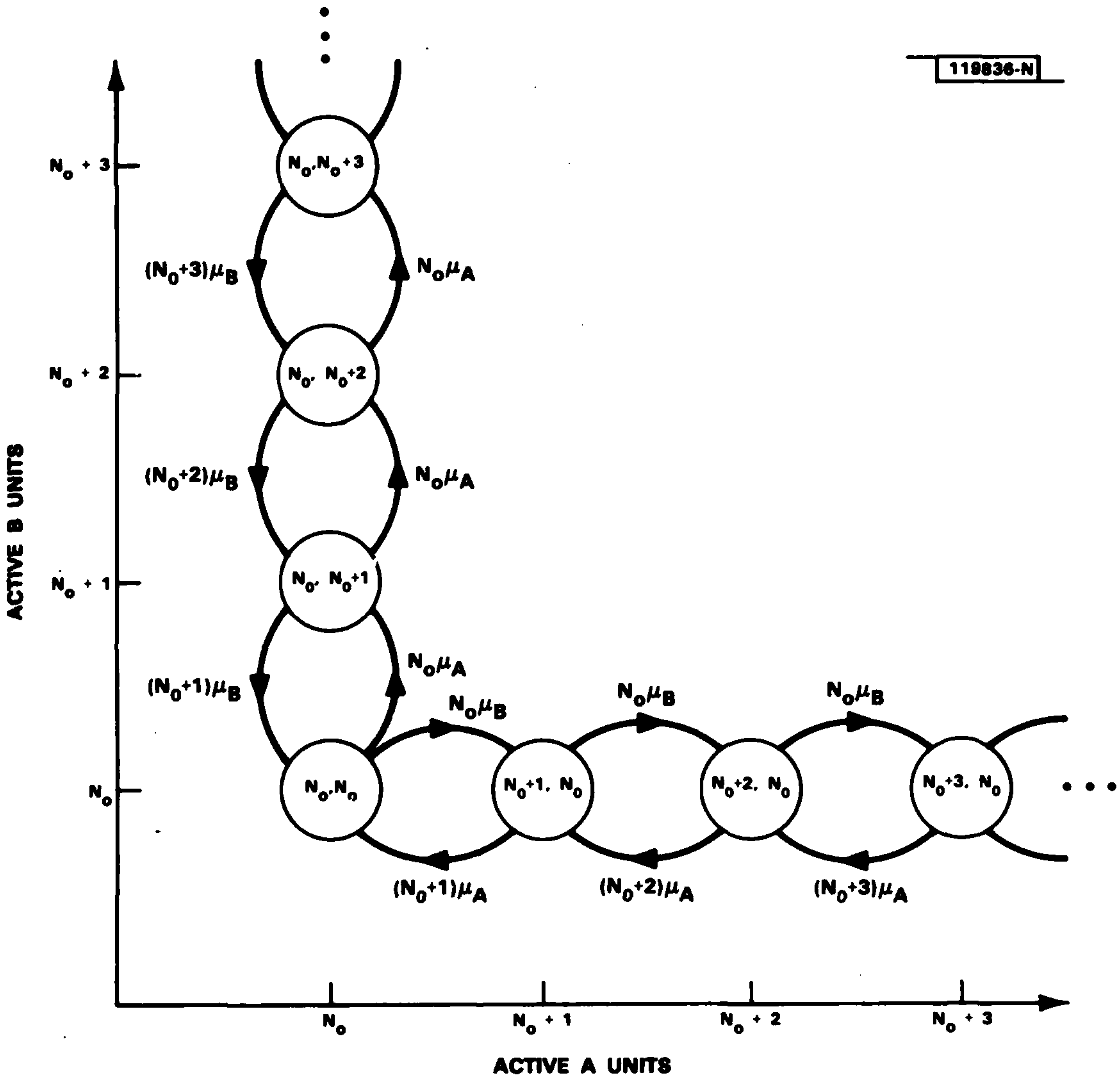


Fig. IV.2. State diagram representing independent failures, replenishment two-at-a-time.

(These expressions do not appear to reduce to more elemental forms.) The value of $P(N_o, N_o)$ is found by requiring that the state probabilities sum to unity.

Once the state probabilities are known then other important questions can be answered. For example, the average required replenishment rate will be:

$$\lambda = N_o \mu_B \sum_{j=0}^{\infty} P(N_o + j, N_o) + N_o \mu_A \sum_{k=0}^{\infty} P(N_o, N_o + k) \qquad \text{(IV-3)}$$

which is found by examining the conditional launch rate for each state. The average number of A units exceeding the required value of N_o will be:

$$\text{average "excess" A's} \triangleq E_A$$

$$= \sum_{j=0}^{\infty} j\, P(N_o + j, N_o) \qquad \text{(IV-4)}$$

with a similar expression for the average excess B's, E_B.

The case of equal failure rates will be examined next as a special case.

1. Equal Failure Rates.

First, let $\mu_A = \mu_B = \mu$. Figure IV-3 gives some numerical results for four parameters:

a. $\lambda/\mu N_o$ is the normalized average replenishment rate from Eq. IV-3. Recall that each replenishment consists of an A and B unit. If units were replaced individually the replenishment rate would be $N_o \mu$ (See Eq. III-1) for each. It can be seen from the figure that this ideal is asymptotically reached only for large N_o. When N_o is moderate, around 5 for example, the average rate is 1.2 $N_o \mu$ which is an increase of 20% over the minimum possible. This is due to the fact that excess units of one type must be placed in service because a circuit of the other type has failed.

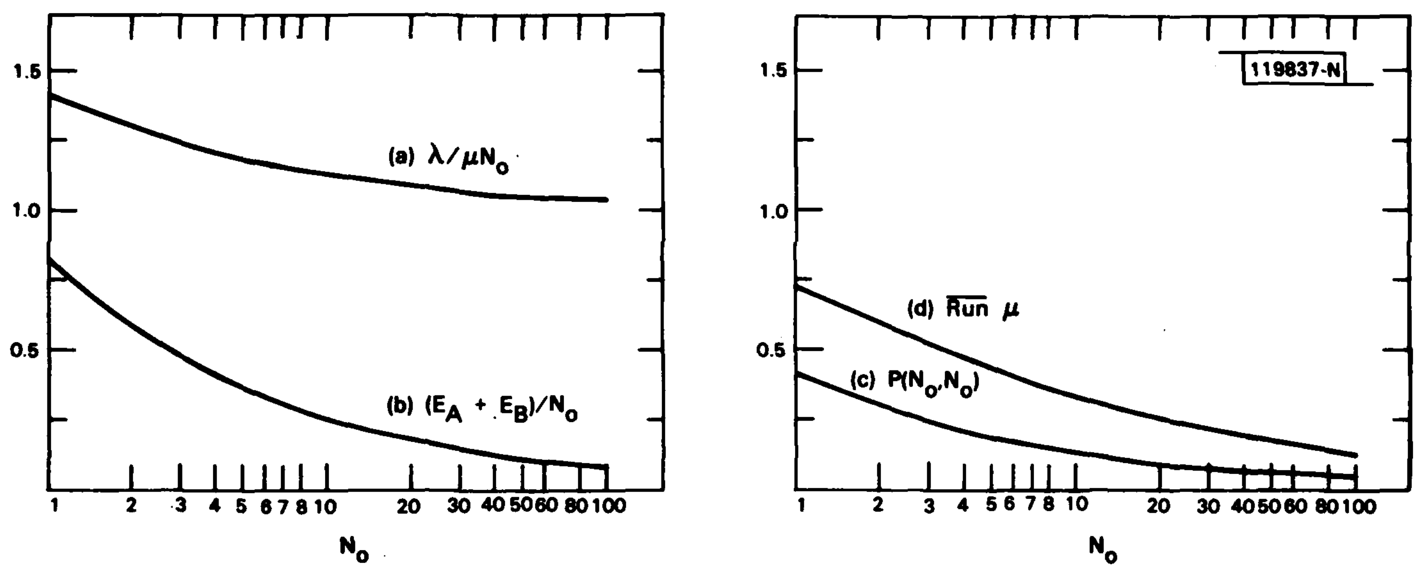


Fig. IV.3. Results for dual replenishments of 2 systems with independent unit failures (equal failure rates): (a) Required replenishment rate, (b) Average excess units, (c) Probability both are critical, (d) Average run length.

b. $(E_A + E_B)/N_o$ is the normalized expected number of "excess" units of either type in service from Eq. IV-4. Only one unit will have excesses at any particular time so this figure is the average number that would actually be observed. (Over the systems' lifetimes, each would exhibit one half of the excesses.) The normalized excess is seen to decrease with N_o. However for moderate $N_o \approx 4$ or 5, there will be about 40% excess units of one type or the other found in service.

c. $P(N_o, N_o)$ is the probability that both systems are "critical", i.e., they both have only N_o units in service. When in this state the average time to next replenishment is $1/(2N_o\mu)$, whereas in all others it is $1/N_o\mu$. The figure shows that the fraction of time in this state decreases with N_o.

d. (Run)μ is the normalized average time the system will have a run of either excess A's or B's. (It is normalized by the unit lifetime, $1/\mu$.) This is described as follows: Suppose the system is in the (N_o, N_o) state with A and B critical. If an A unit were to fail, the replenishment needed would put B in excess. The system will remain with B in excess until the next time (N_o, N_o) is reached. This time in excess is being called a run. It gives an indication of how long it takes to flush excess units out of the system.* The figure shows that the average run time decreases with N_o and is always less than the lifetime of a single unit, but may last for many average replenishment times. It is found as follows:

At any time, the system will either be in state (N_o, N_o) or in a run. The fraction of time the system is in state (N_o, N_o) is just $P(N_o, N_o)$; the fraction of time spent in all runs is $1-P(N_o, N_o)$. Each time the system leaves (or enters) state (N_o, N_o) a run is begun (or ended). Thus if the system is observed over some long time period T the expected total time spent in (N_o, N_o) can be written as:

* Note that the average number of excess units during a run is equal to E_A or E_B divided by the probability of being in an excess of A or B run respectively.

expected total time in (N_o, N_o) (IV-5)

= (expected # of runs) (expected time to be held in (N_o, N_o))

Two of the three terms can be substituted with other expressions:

$$T\ P(N_o, N_o) = (\text{expected \# of runs})\ (1/2N_o\mu) \tag{IV-6}$$

Thus,

$$\text{expected \# of runs} = 2N_o\mu T\ P(N_o, N_o) \tag{IV-7}$$

Since

expected total time in runs

= (expected # of runs) (expected run length)

$$= T(1-P(N_o, N_o)) \tag{IV-8}$$

it is found that

expected run length $\triangleq \overline{\text{Run}}$

$$= \frac{1- P(N_o, N_o)}{2N_o\mu\ \ P(N_o, N_o)} \tag{IV-9}$$

The curve of Figure IV-3d results.

2. Unequal Failure Rates

If one of the units is less reliable then it will have a higher failure rate. Suppose A is less reliable so that

$$\mu_A > \mu_B.$$

Figure IV-4 gives some numerical results illustrating the effects of imbalanced failure rates. The curves are plotted versus the ratio of failure rates μ_A/μ_B for a few values of N_o.

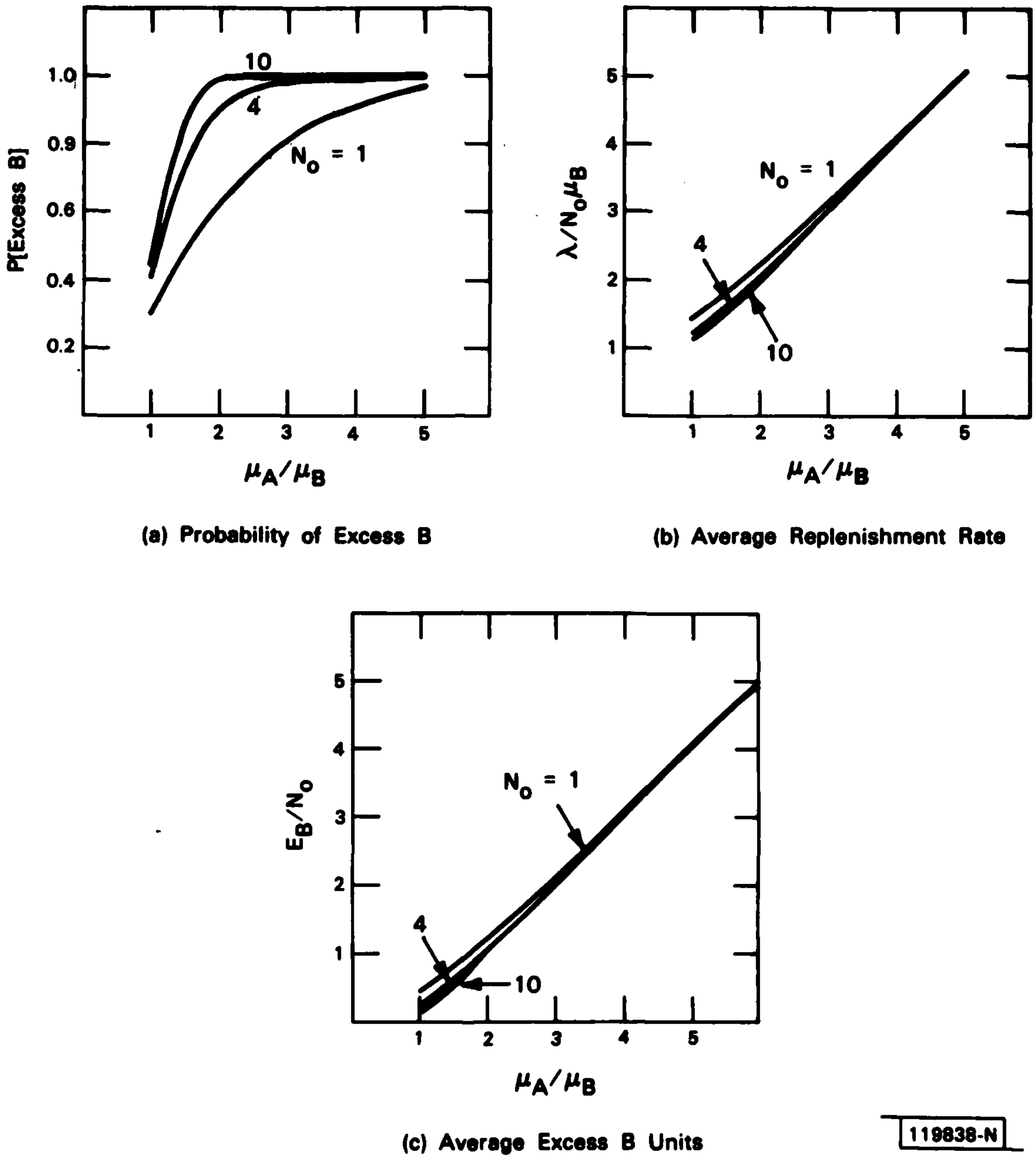


Fig. IV.4. Results for dual replenishment with independent failures (unequal failure rates).

a. P[Excess B] is the probability that there is at least one excess B. As μ_A/μ_B increases this probability approaches unity since the less reliable A's are forcing unnecessary replenishments of B units.

b. $\lambda/N_o\mu_B$ is the replenishment rate normalized by the failure rate of B's. When μ_A/μ_B is large enough the replenishment rate will be dominated by the failures in the A units and λ will approach $N_o\mu_A$. Thus $\lambda/N_o\mu_B$ will approach μ_A/μ_B as shown in the figure.

c. E_B/N_o is the expected number of excess B units normalized by N_o. The expected number of B units in service is λ/μ_B. As μ_A/μ_B increases this will approach $N_o(\mu_A/\mu_B)$. Hence the expected normalized excess number of B units will approach (μ_A/μ_B-1) as indicated. The number of excess A units approaches zero since they are almost always in critical supply.

B. Including Satellite Bus Failures.

Figure I-1 illustrated the point that satellites are constructed with the payload units mounted on a bus. In addition to the payload unit failures which were treated in the last section, the effects of bus failures will now be included. At any time, satellites with three combinations of working units may be found on-orbit:

<u>Type AB</u> with both payloads and the bus operating.
<u>Type A</u> with only payload A and the bus operating (B having failed)
<u>Type B</u> with only payload B and the bus operating (A having failed)

(In addition, of course, there would be useless dead-wood consisting of satellites with failed buses and/or both A and B packages that have failed.)

The state of the system can be characterized by a triplet of numbers indicating the number of each type of useful satellite remaining. An enumeration of the states as well as the transition rates among them is shown in Figure IV-5 in which the desired number of working units, N_o, is equal to 2. Each state is labeled with the number of type AB, type A, and type B

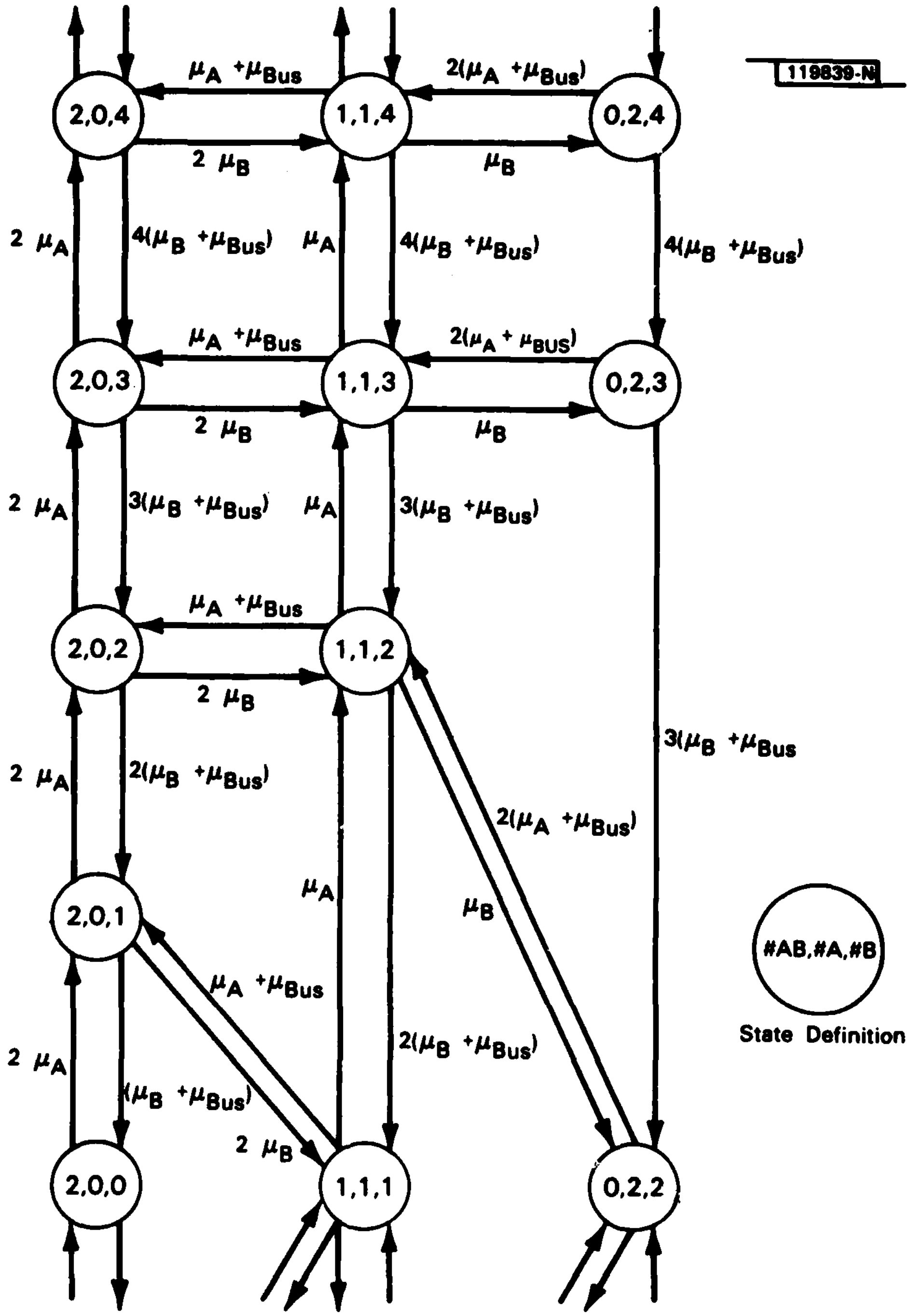


Fig. IV.5. State diagram including bus failures (N_o = 2 active units required).

satellites. The transition rates are given in terms of the package failure rates, μ_A and μ_B, as well is the bus failure rate, μ_{Bus}.* It is assumed that failures of the packages and buses occur independently. The figure shows only the upper half of the state diagram, including just those states with both systems critical or excess B's (equivalent to those states with exactly 2 working A's). It should be clear how to extend it symmetrically to excess A's.

Figure IV-6 is an annoted example of the transitions from one of the states, (1, 1, 2), also showing which transitions are concommitant with a replenishment launch (not all of them are). It should also be noted that there is a self-loop that occurs if the bus of the type AB satellite fails. The state would not change but a replenishment would be required. This occurs at rate μ_{Bus}.

The equilibrium state probabilities fo the system shown in Figure IV-5 were found using standard Markov chain techniques [Ref. 1, Section 2.4]. The approach is given briefly as follows:

First number the states (in any order). Then define the elements q_{ij} of the matrix transition rates Q as follows:

$$\begin{aligned} q_{ij} &= \text{transition rate from state } i \rightarrow \text{state } j \quad i \neq j \\ q_{ii} &= -\sum_{j \neq i} q_{ij} \end{aligned} \tag{IV-10}$$

Then putting the state probabilities P_j into row-vector form, P, the following linear matrix equation and normalizing condition will yield the solution for P:

* However another interpretation of bus failure is that of an entire satellite failure, or failure of function needed by the other packages, e.g., a cross-link, or that of non-independent package failures. The appropriate interpretation depends on the application.

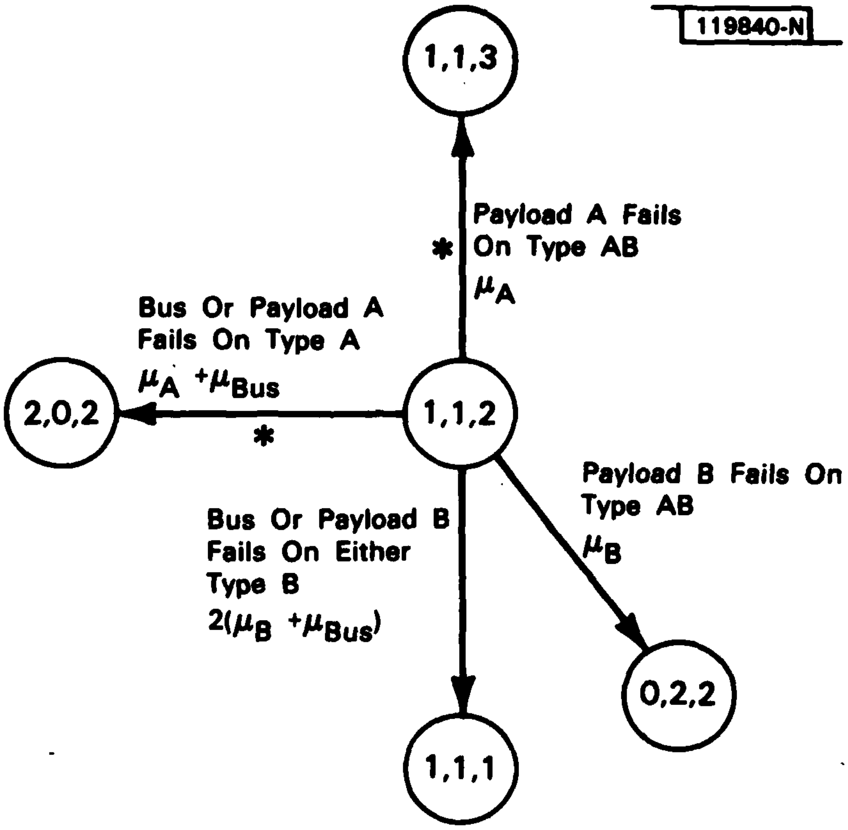


Fig. IV.6. Annotated transition rates from state (1, 1, 2). (* indicates replenishment required.)

$$PQ = 0$$

and (IV-11)

$$\sum_j P_j = 1$$

The matrix equation is nothing more than the equilibrium "rate-in equals rate-out" condition for each of the states.

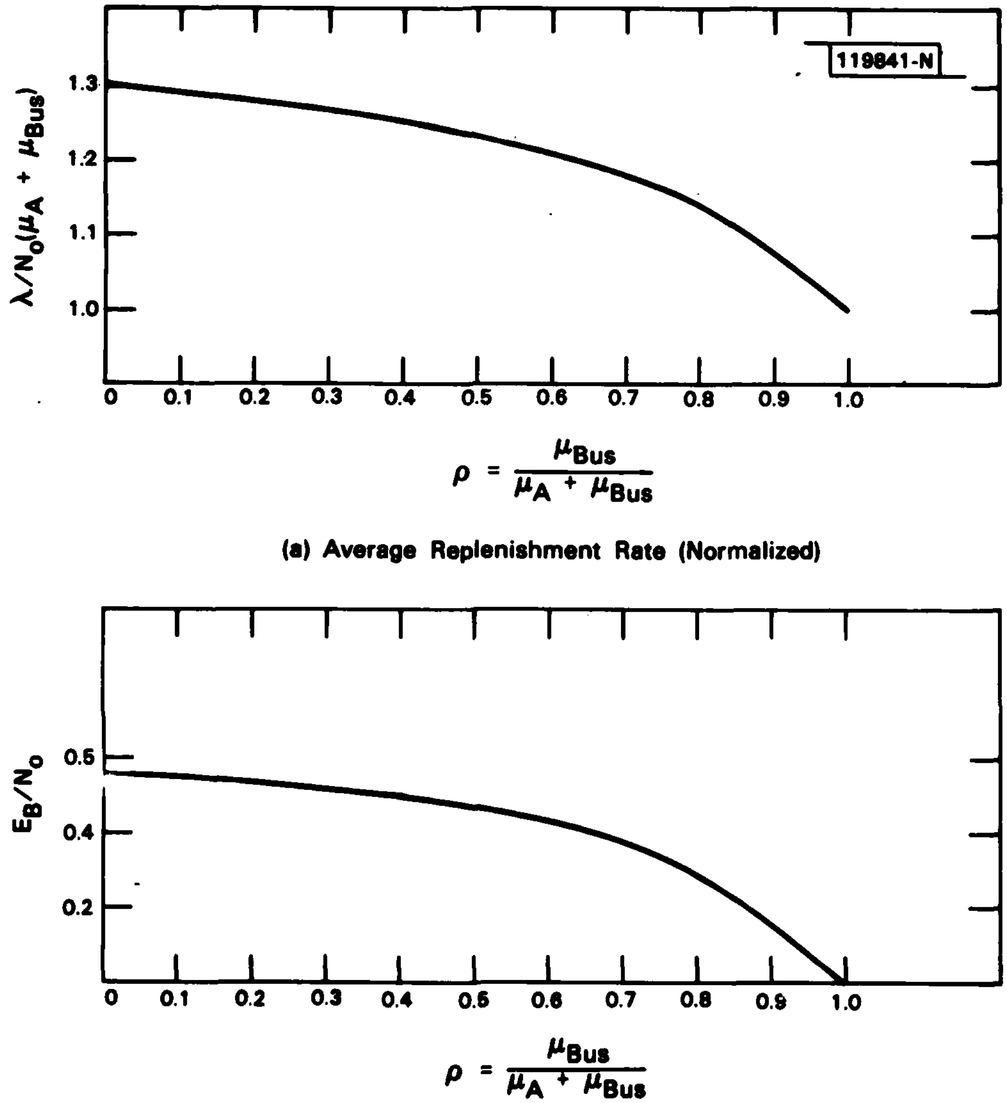
These equations were solved for the system of Figure IV-5 by first truncating the number of states to those shown plus the symmetrical lower ones. Only the case $\mu_B = \mu_A$ was considered. The probability of reaching the extreme states turned out to be less than .05, showing that the truncation did not significantly disturb the results to follow.

Figure IV-7 shows two key numerical results. Both are plotted versus the parameter $\rho = \mu_{Bus}/(\mu_A + \mu_{Bus})$ which is an indication of the relative failure rates of the bus and each package. When $\rho=0$ it is equivalent to no bus failures, the case treated in the previous section. When equal to one, the only failures are bus failures (or simultaneous package failures).

a. $\lambda/N_o(\mu_A + \mu_{Bus})$ is the normalized expected replenishment rate. $\mu_A + \mu_{Bus}$ can be thought of as the mission failure rate for each payload caused by either a payload or bus failure. When $\rho=0$ the result is the same as the previous section's (see Figure IV-3a). When $\rho=1$ the replenishment rate exactly matches the bus failure rate, $N_o\mu_{Bus}$, which will imply 100% effective replenishment of failed resources with no build up of excesses.

λ is calculated by examining the conditional launch rate of each state, weighting it by the state's probability and summing. The conditional launch rate for all states where only A is critical is $2(\mu_A + \mu_{Bus})$. The launch rate for those states where A and B are both critical ((2, 0, 0), (1, 1, 1) and (0, 2, 2)) is $2\mu_A + 2\mu_B + (\text{\# of buses})\mu_{Bus}$.

b. E_B/N_o is the expected normalized excess number of payloads of either type that would be found. As shown it is maximum when $\rho=0$, indicating



(a) Average Replenishment Rate (Normalized)

(b) Average Excess (Normalized)

Fig. IV.7. Results for dual replenishment including bus failures (equal package failure rates). $N_o = 2$.

independent failures allowing excesses of one type or another to build up. When $\rho=1$ no excess units build up because all failures are double failures, replenished efficiently by just the right number of new units.

The reader should see how to generalize to other values of N_o or differing μ_A and μ_B: It is a question of careful bookeeping to account for the allowed state transitions. The qualitative nature of the results should also be clear.

It also should be noted that the transient behavior can be found. In particular if the state of the system at time $t=0$ is known, the probability row vector at time t will be:

$$P(t) = P(0)e^{Qt} \tag{IV-12}$$

where P(0) is the probability row vector at $t=0$. (The usual definition of matrix exponentiation is being used.)

V. REPLENISHMENT AS-NEEDED, S=3 SYSTEMS

When dealing with S=3 systems, (labeled A, B and C), the units can be replenished one, two or three at-a-time. (Only replenishment as-needed strategies are treated here. Scheduled replenishment was treated in Section II.)

If replenished one at a time, then according to Eq. (III-1), the replenishment rate will be equal to $N_o \mu$, where N_o = the number of active units maintained and μ = the failure rate per unit. (These parameters need not be the same for each system.)

Replenishing two-or-three-at-a-time is considerably more involved. While Markov chain models can be developed that take into account bus failures, differing unit failure rates, differing active unit requirements and differing strategies for deciding which excess units should be added, only some simplified cases will be examined here. The simplifying assumptions to be used are:

1.) the same number of active units are required in each system and will be denoted as N_o.

2.) the failure rate per unit is the same for each system and will be denoted as μ

3.) unit failures occur independently and with exponential distribution for their lifetime (average lifetime = $1/\mu$ per unit).

4.) no "bus" failures are considered.

5.) replenishing three-at-a-time always includes one of each type of unit, e.g., only ABC never ABB.

and 6.) replenishing two-at-a-time always includes units of two different types, e.g., AB is allowed but AA is not.

With these assumptions, the three-at-a-time and two-at-a-time solutions will now be presented.

A. Replenishing Three-at-a-Time

When replenishing three-at-a-time, new units A, B and C will be placed in service when the number of units of any one system drops below N_o. While the state of such a system could be taken as the number of A, B and C units in service there is another representation that requires fewer states. It is sufficient to know the count of the three types of units without regard to which unit is associated with which count. For example if one system has N_o another has $N_o + 1$ and the third $N_o + 3$, this state specification would actually represent any of the 6 ways to assign these counts to systems A, B, C. This state will be labeled (0, 1, 3) indicating the number of units in excess of N_o. (One of the terms must always be zero. Even though superfluous, it will be carried along as a reminder.) The counts will always be shown in non-decreasing order. This simplification will be valid because the failure rates for the units are the same as are the number of required active units for each system.

Figure V-1 shows the Markov chain model using the state definition just described. The transition rates are shown, omitting the factor of μ that multiplies each value. The transitions marked with an asterisk are those associated with a replenishment. The conditional replenishment rate for each state is $N_o\mu$ times the number of counts that are at zero excess.

In order to solve this probabilistic system it was turned into a finite Markov chain by allowing up to only M excess units of any type. Any units in excess of M are permanently lost. For M=3 this is equivalent to redirecting the downward transitions from the bottom row of Fig. V-1 to the left (and eliminating the downward transition from (0, 3, 3)). Then the procedure outlined in Section IV could be applied. After solving for the state probabilities the replenishment rate was calculated and it is shown in Figure V-2 normalized by $N_o\mu$ versus N_o for values of M from 3 to 6. The replenishment rate is that for each unit to facilitate comparisons with the one or two-at-a-time cases. In the three-at-a-time case the package replenishment rate is equal to the bulk replenishment rate since each replenishment contains one of each type of unit.

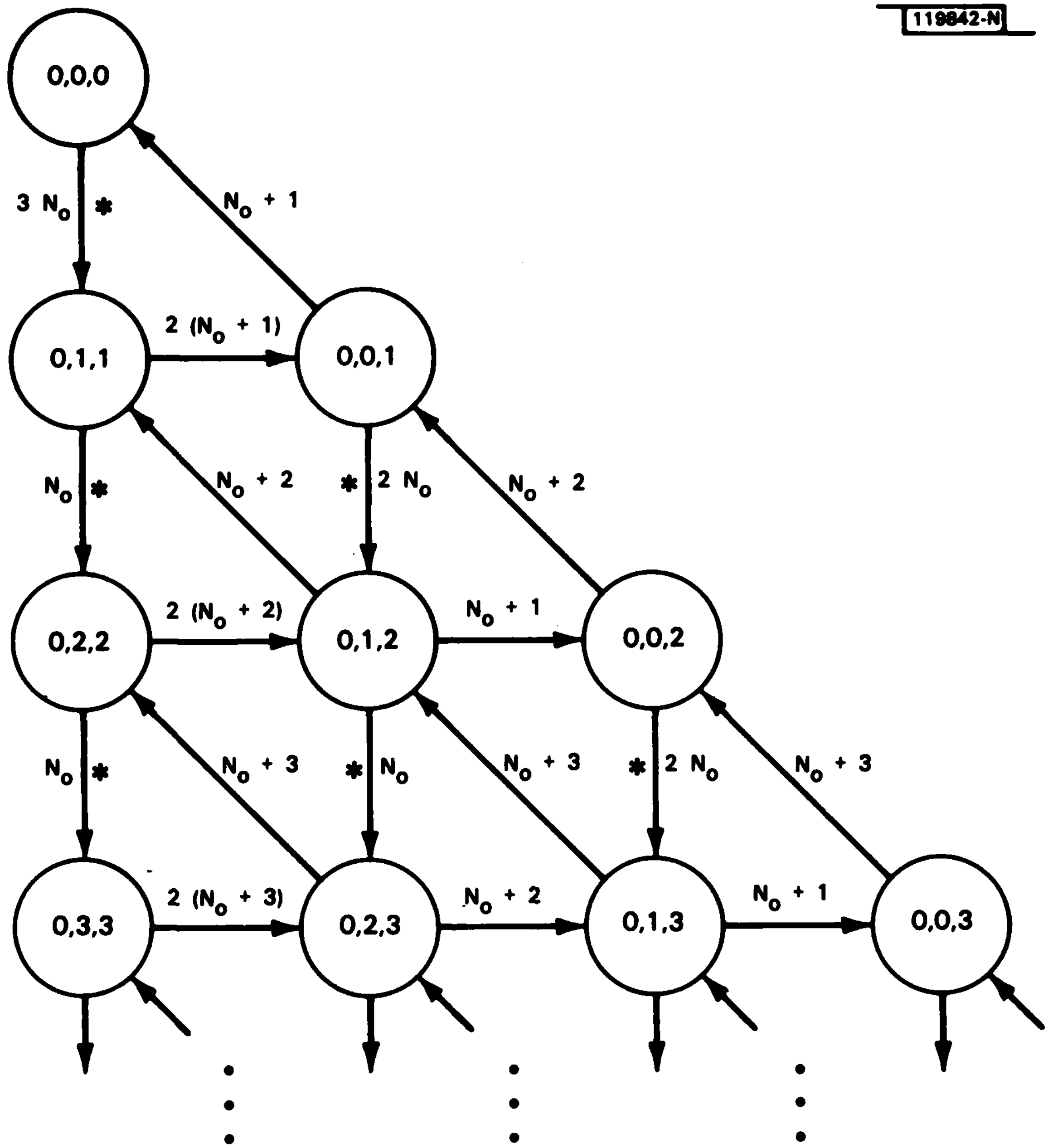


Fig. V.1. Model representing replenishment of 3 systems three-at-a-time. (Rates shown are normalized. States show excess active units.)
* = launch required.

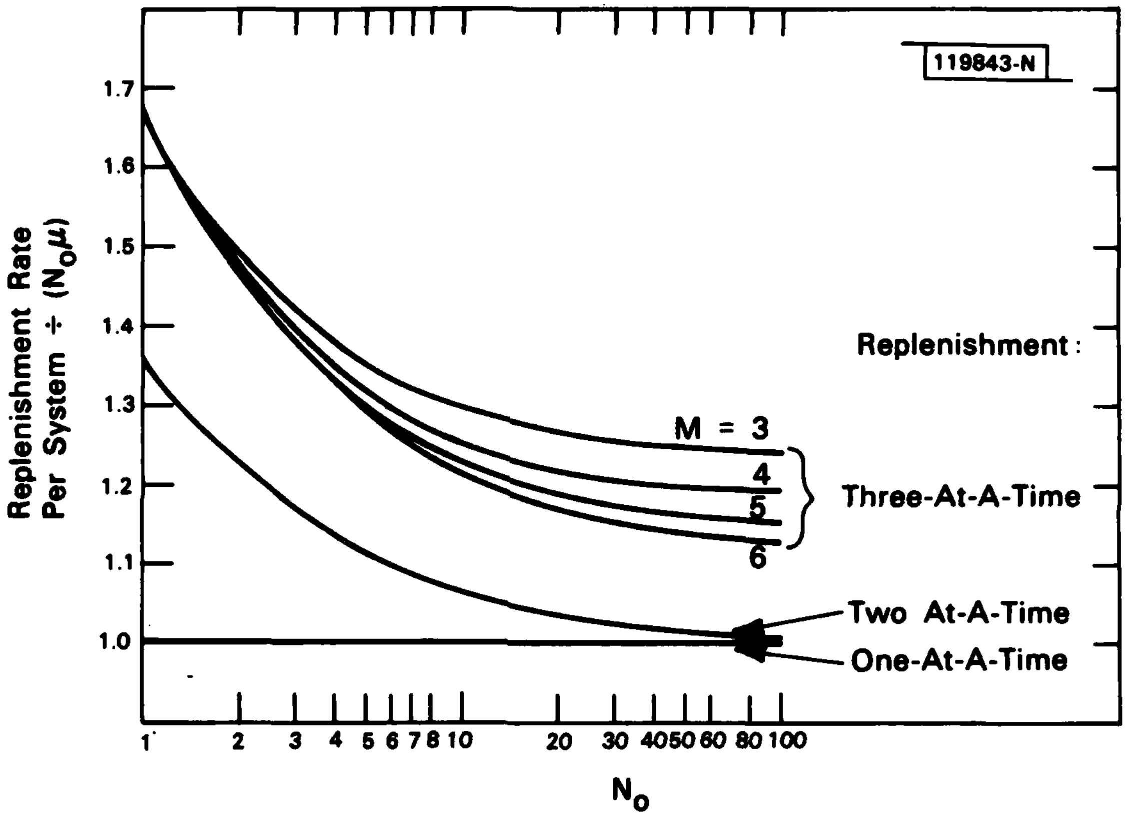


Fig. V.2. Normalized replenishment rate per system for 3 systems.

The curves show that as N_o grows, the replenishment process becomes more efficient with the normalized rate heading toward its minimum possible value of unity. (This is achieved only with one-at-a-time replenishment.) Also, as the number of allowed excess units increases the replenishment efficiency becomes better because there are fewer wasted spares. For N_o up to about 10 the numerical results show that there will be little reduction in the required replenishment rate even if larger M were allowed. For example, for N_o=5 the probability of ever having 6 or more active spares of any unit will be less than .05 showing that increasing M beyond 6 will have little effect.

B. Replenishing Two-at-a-Time

The two-at-a-time curve in Figure V-2 was found in a similar manner from the state diagram of Figure V-3. The state notation is the same as Figure V-1. The replenishment is as follows: If the unit count for any system drops below the critical value of N_o a replenishment is performed consisting of a replacement for the failed unit plus a unit of the system remaining with the lower count.* For example suppose there are N_o A-units, N_o+ 1 B-units and N_o+2 C-units (state (0, 1, 2)). If a B or C unit fails no replenishment is required. However if an A unit fails a replenishment will be arranged with an A and B unit sending the system to state (0, 2, 2). This transition happens with rate $N_o\mu$.

The bulk replenishment rate is calculated in an identical way to the three-at-a-time strategy. This is the rate of required replenishments of any type. In satellite terminology this would be the required launch rate. For the two-at-a-time strategy, however, the replenishment rate per system will be 2/3 of the bulk rate, since on the average each unit type will appear on on only 2/3 of the replenishments. The replenishment rate per system is shown in Figure V-2.

The calculation was performed allowing a maximum of 4 spare (above N_o) units. However the results showed that, even with N_o=100 units required, the

* If the two remaining systems have the same count a random choice is made.

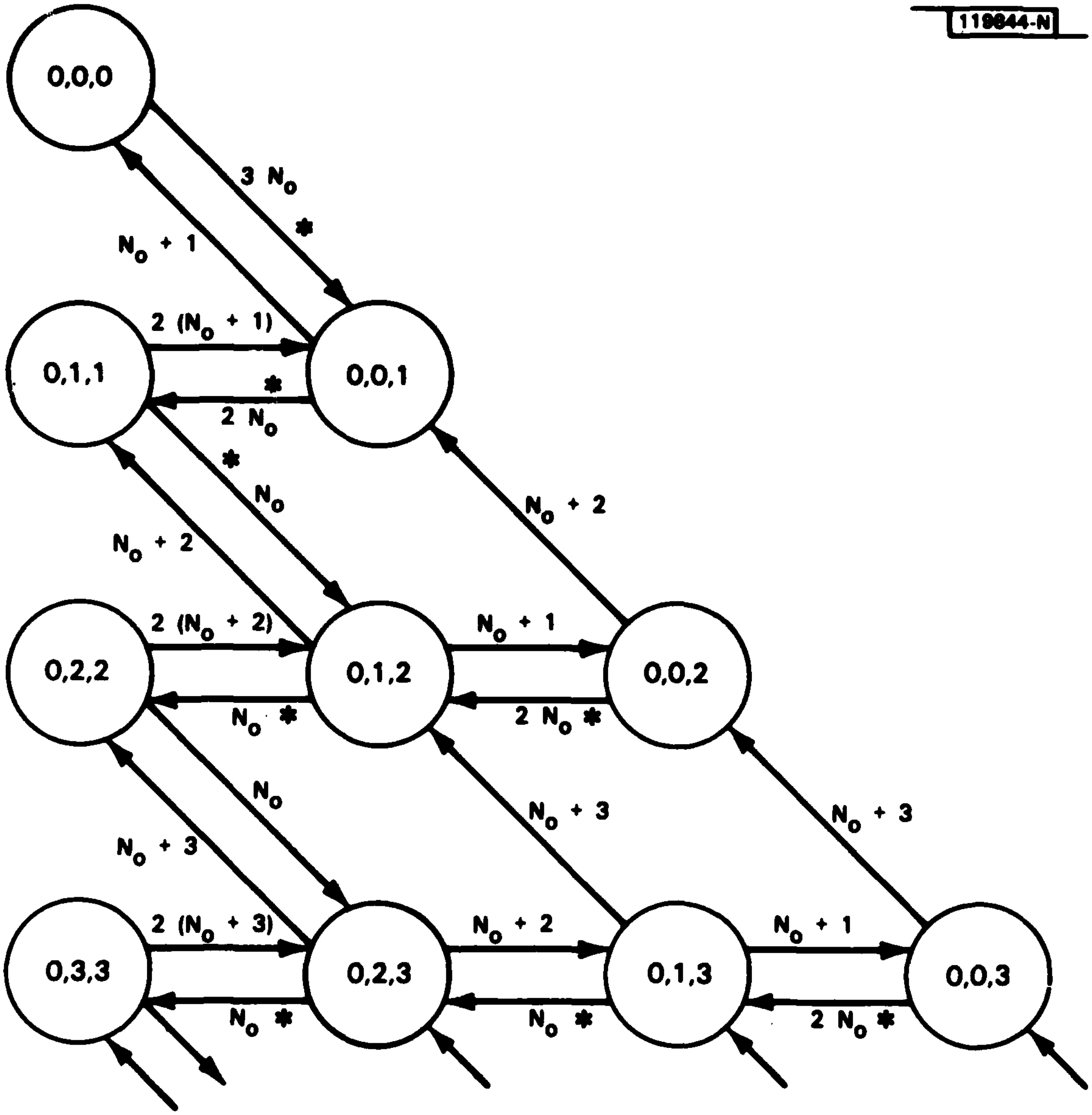


Fig. V.3. Model representing replenishment of 3 systems two-at-a-time. (Rates shown are normalized. States show excess active units.) * = launch required.

probability of having 4 or more spare units is less than .02. Thus the two-at-a-time curve appears valid even if unlimited spares were allowed.

Once again it is seen that with larger N_o required the resulting replenishment is done with greater efficiency. It is also clear that two-at-a-time replenishment is more efficient than three-at-a-time replenishment. This is expected since assets are being placed into service with a distribution more closely matching the failure patterns. The most efficient is, of course, to replenish only the units that have failed one-at-a-time.

It is important to remember that replacement rate is not the only criterion on which to base a replenishment strategy. It is the overall cost that must be considered. In particular, for satellite applications, the cost of replacing a payload with a single mission satellite could be more than 1/2 or 1/3 of the cost of two-at-a-time or three-at-a-time replenishment because of shared overhead functions for multi-mission satellites. On the other hand the mission failure rates (bus plus package) for multi-mission satellites may be greater due to increased complexity.

C. Bus Failures.

While no detailed analysis of the type done in Section IV-B for 2 systems including bus failures has been done, there are some observations that can be made. Suppose that the only failures were due to bus failures. Then the bulk replenishment (launch) rates would be as follows:

$$\lambda = \begin{cases} 3N_o \mu_{Bus}(1) & \text{one-at-a-time} \\ \frac{3N_o}{2} \mu_{Bus}(2) & \text{two-at-a-time} \\ N_o \mu_{Bus}(3) & \text{three-at-a-time} \end{cases}$$

whre $\mu_{Bus}(k)$ = bus failure rate for k-at-a-time replenishments. The launch rate <u>per system</u> normalized by the mission failure rate ($\mu_{Bus}(k)$) and the number of units required all turn out to be the same, unity.

The greatest spread in replenishment rates (normalized by mission failure rates) among the three strategies will be for the case of independent (no bus) failures and equal unit failure rates. This is shown in Figure V-2.

VI. CONCLUSIONS

Both scheduled and as-needed replenishment strategies were analyzed for multiple resource systems. It was found that in general the most efficient strategies are those for which the replenishment is made with a combination of units most closely resembling the failed resources. In this context, "most efficient" refers to the smallest expected unit replenishment rate required to guarantee a given level of continuing service. Quantitative results were given for many cases of interest. The techniques used can be extended to other replenishment strategies and failure models.

The results are particularly applicable to multi-mission satellite systems and form a major element in the economic analysis of such systems.

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Both scheduled and replenishment as-needed strategies are analyzed for multiple resource systems. It is found that in general the most efficient strategies are those for which the replenishment is made with a combination of units most closely resembling the failed resources. In this context, "most efficient" refers to the smallest expected unit replenishment rate required to generate a given level of continuing service. Quantitative results are given for many cases of interest including replenishment with multiple resources of different types. The techniques used can be extended to other replenishment strategies and failure models.

The results are particularly applicable to multi-mission satellite systems and can contribute to the economic analysis of such systems.

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